14.7 Apply Double-Angle and Half-Angle Formulas

$\boxed{\cos^2 a+\sin^2 a = 1}$

Goal • Use double-angle and half-angle formulas.

$\cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1-2\sin^2 a$

$\tan a = 2*t$

$\frac{}{1-tar}$

Your Notes

DOUBLE-ANGLE AND HALF-ANGLE FORMULAS

Double-Angle Formulas	**Half-Angle Formulas**
$\boxed{\cos 2a = \cos^2 a - \sin^2 a}$	$\sin \frac{a}{2} = \sqrt{\frac{1-\cos a}{2}}$
$\cos 2a = 2\cos^2 a - 1$	$\cos \frac{a}{2} = \sqrt{\frac{\cos a+1}{2}}$
$\cos 2a =$ ___ $-$ ___	$\tan \frac{a}{2} =$ ___
$\sin 2a =$ ___	$\tan \frac{a}{2} = \sqrt{\frac{1-\cos a}{1+\cos a}}$
$\tan 2a =$ ___	

$\tan(a+b) =$

$\frac{2\tan a}{1-\tan^2 a}$

Example 1 ***Evaluating trigonometric expressions***

Find the exact value of $\cos \frac{\pi}{8}$.

Solution

$$\cos \frac{\pi}{8} = \cos \frac{1}{2}\left(\frac{\pi}{4}\right) = \sqrt{\frac{1+\cos \frac{\square}{4}}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} \quad \sqrt{\frac{\frac{2+2\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+2\sqrt{2}}{4}}$$

$$= \frac{\sqrt{1+\sqrt{2}}}{2}$$

Because $\frac{\pi}{8}$ is in Quadrant I and the value of cosine is positive in Quadrant I, the following formula is used:

$\cos \frac{a}{2} = \sqrt{\frac{1+\cos a}{2}}$

Your Notes

In part (b), you can multiply through the inequality $\frac{\pi}{2} < a < \pi$ by $\frac{1}{2}$ to get $\frac{\pi}{4} < a < \frac{\pi}{2}$. So, $\frac{a}{2}$ is in Quadrant I.

Example 2 *Evaluate trigonometric expressions*

Given $\sin a = \frac{3}{5}$ with $\frac{\pi}{2} < a < \pi$, find (a) $\cos 2a$ and (b) $\cos \frac{a}{2}$.

Solution

Using a Pythagorean identity gives $\cos a =$ ____.

a. $\cos 2a =$ __________ $- 1 =$ ____$\left(____\right)^2 - 1 =$ ____

b. Because $\frac{a}{2}$ is in Quadrant I, $\cos \frac{a}{2}$ is __________.

$\cos \frac{a}{2} =$ ____________ $=$ ____________ $=$ ______

Checkpoint Complete the following exercises.

1. Find the exact value of $\sin \frac{\pi}{12}$.

2. Given $\cos a = -\frac{7}{25}$ with $\pi < a < \frac{3\pi}{2}$, find $\sin 2a$.

Example 3 *Verify a trigonometric identity*

Verify the identity $\sin 4x = 4 \sin x \cos x(1 - 2 \sin^2 x)$

Solution

$\sin 4x = \sin(2x +$ ____$)$

$= \sin 2x \cos 2x +$ _______________

$= (2 \sin x \cos x) \cos 2x + \cos 2x($_______________$)$

$= (2 \sin x \cos x + 2 \sin x \cos x)$ _________

$= (4 \sin x \cos x)$ _________

$= 4 \sin x \cos x($______________$)$

Your Notes

Example 4 *Solve a trigonometric equation*

Solve $\cos 2x + \sin x = 0$ for $0 \le x < 2\pi$.

Solution

$$\cos 2x + \sin x = 0$$

$$\_\_\_\_\_\_\_\_ + \sin x = 0$$

$$\_\_\_\_\_\_\_\_ + \sin x + \_\_\_ = 0$$

$$(\_\_\_\_\_\_ + \_\_\_)(\_\_\_\_\_\_ + \_\_\_) = 0$$

$\_\_\_\_\_\_ + \_\_\_ = 0$ or $\_\_\_\_\_\_ + \_\_\_ = 0$

$\_\_\_\_\_\_ = -1$ $\_\_\_\_\_\_ = -1$

$\sin x = \_\_\_$ $\sin x = \_\_\_$

$x = \_\_\_, \_\_\_$ $x = \_\_\_$

Checkpoint **Complete the following exercises.**

3. Verify the identity $\cos 3x = \cos^3 x - 3 \sin^2 x \cos x$.

4. Solve $\tan \frac{x}{2} = \sin x$ for $0 \le x < 2\pi$.

Homework

$\cos\frac{a}{2} =$ ______________

$\sin^2 b + \cos^2 b = 1$ $\quad b = a/2$

$\cos^2 b = 1 - \sin^2 b$

$\cos b = \sqrt{1 - \sin^2 b}$ $\checkmark$

$\sin\frac{a}{2} = \boxed{\sqrt{1 - \cos^2\frac{a}{2}}}$

$\boxed{\cos\frac{a}{2} = \sqrt{1 - \sin^2\frac{a}{2}}}$

$\tan\frac{a}{2} \quad \frac{\sin\frac{a}{2}}{\cos\frac{a}{2}} \quad \frac{\boxed{}}{\boxed{}}$

$$\cos\frac{a}{2} = \sqrt{\frac{\cos a+1}{2}}$$

$$\sin\frac{a}{2} = \sqrt{\frac{1-\cos a}{2}} \qquad \tan\frac{a}{2}$$

$$\cos 2a = \cos^2 a-\sin^2 a = 2\cos^2 a-1 = 1-2\sin^2 a$$

$$\boxed{\cos a} = \qquad = 2\boxed{\cos^2 a/2}-1 = \boxed{1-2\sin^2\frac{a}{2}}$$

$$\cos a+1 = 2\cos^2\frac{a}{2}$$

$$\cos a = 1-2\sin^2\frac{a}{2}$$

$$\boxed{\sqrt{\frac{\cos a+1}{2}}}=\cos^{\cancel{2}}\frac{a}{2}$$

$$2\sin^2\frac{a}{2} = 1-\cos a$$

$$\sin\frac{a}{2} = \sqrt{\frac{1-\cos a}{2}}$$